Mathematical Analysis - List 20

1. Compute the partial sums and determine whether the series is convergent or divergent.

a) $\sum_{n=1}^{\infty} \frac{1}{4n^2-1}$; b) $\sum_{n=1}^{\infty} \frac{n-1}{n!}$;

c) $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n+1}+\sqrt{n}}$; d) $\sum_{n=1}^{\infty} \left[\sin\left(\frac{1}{n}\right) - \sin\left(\frac{1}{n+1}\right)\right]$.

2. If the nth partial sum of a series $\sum_{n=1}^{\infty} a_n$ is $s_n = 3 - n2^{-n}$ find a_n and $s = \sum_{n=1}^{\infty} a_n$.

3. Use the Integral test to determine whether the series converges or diverges.

a) $\sum_{n=1}^{\infty} \frac{1}{n^2+n}$; b) $\sum_{n=1}^{\infty} \frac{n}{n^2+4}$;

c) $\sum_{n=2}^{\infty} \frac{\ln n}{n^2}$; d) $\sum_{n=1}^{\infty} \frac{1}{n\sqrt{n+1}}$.

4. Find the values of p for which the following series is convergent $\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^p}$.

5. Use the Comparison test to determine whether the series converges or diverges.

a) $\sum_{n=1}^{\infty} \frac{3}{n^2+2}$; b) $\sum_{n=1}^{\infty} \frac{n+1}{n^2+1}$; c) $\sum_{n=1}^{\infty} \sin\frac{\pi}{2^n}$.

d) $\sum_{n=0}^{\infty} \frac{2^n+\sin n!}{3^n}$; e) $\sum_{n=1}^{\infty} \frac{3-2\cos n^2}{\sqrt{n}}$; f) $\sum_{n=1}^{\infty} \frac{3^n+1}{n3^n+2^n}$.

6. Use the Limit Comparison test to determine whether the series converges or diverges.

a) $\sum_{n=1}^{\infty} \frac{3^n+15}{4^n-13}$; b) $\sum_{n=1}^{\infty} \frac{n+1}{\sqrt{n^3+1}}$; c) $\sum_{n=1}^{\infty} \frac{\sin\frac{\pi}{3^n}}{\sin\frac{\pi}{2^n}}$.

7. Use the Ratio (d'Alembert) test to determine whether the series converges or diverges.

a) $\sum_{n=1}^{\infty} \frac{100^n}{n!}$; b) $\sum_{n=1}^{\infty} \frac{(n!)^2}{(2n)!}$; c) $\sum_{n=1}^{\infty} \frac{n^n}{3^n n!}$.

8. Use the Cauchy test to determine whether the series converges or diverges.

a) $\sum_{n=1}^{\infty} \frac{2^n+3^n}{3^n+4^n}$; b) $\sum_{n=1}^{\infty} \frac{(n+1)^{2n}}{(2n^2+1)^n}$; c) $\sum_{n=1}^{\infty} \arccos^n \frac{1}{n^2}$.

9. Use the Alternating Series Test to determine whether the series converges or diverges.

a) $\sum_{n=1}^{\infty} (-1)^n \frac{n-1}{n^2+5}$; b) $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{\ln n}{n \ln\ln n}$; c) $\sum_{n=1}^{\infty} (-1)^{n+1} \left[e - \left(1+\frac{1}{n}\right)^n\right]$.